

#7 MEASURING SPEED 2.1

1. Several students decide to perform an experiment. They throw one of their calculus books into the air and record its height at various times.

(a) NUMERICAL Data:

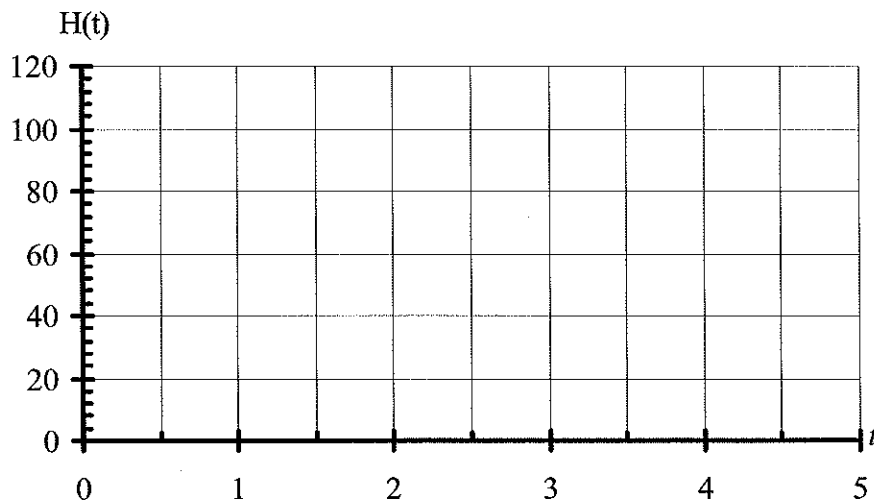
$t = \text{time (sec)}$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$H = \text{height (ft)}$	6.0	44.5	75.0	97.5	112.0	118.5	117	107.5	90.0	64.5	31.0

(b) ALGEBRAIC Interpretation:

The equation that best fits the data above is $H(t) = at^2 + bt + c$. Use the data to find the values of a , b , and c . SHOW ALL WORK.

(c) GEOMETRIC Interpretation:

Use your calculator to graph this equation; then neatly and accurately sketch the graph below:



- (d) Find the average velocity of the book between $t = 1.0$ and $t = 2.5$. SHOW ALL WORK.

$$\text{Note: average velocity over } [a,b] = \frac{\text{change in position}}{\text{change in time}} = \frac{H(b) - H(a)}{b - a}$$



(e) Draw a line on your graph to part (c) that connects the points $(1, H(1))$ and $(2.5, H(2.5))$. Calculate the slope of this line.

(f) Now compute the average velocity of the book between $t = 3.0$ and $t = 5.0$. Why do you think the sign is negative?

BY NOW YOU SHOULD NOTICE THAT THE AVERAGE VELOCITY OVER ANY TIME INTERVAL $[a,b]$ IS THE SLOPE OF THE SECANT LINE JOINING THE POINTS ON THE GRAPH OF $H(t)$ CORRESPONDING TO $t = a$ AND $t = b$.

2. How would we get the velocity of the book at exactly the instant $t = 2.5$ sec?

To do this we sneak up on $t = 2.5$ by taking values of t close to $t = 2.5$ (such as, $t = 2.49$ and $t = 2.499$ and $t = 2.4999$). This gets us an approximation as we approach $t = 2.5$ from the left.

To get an approximation as we approach $t = 2.5$ from the right, we let $t = 2.51$, $t = 2.501$, $t = 2.5001$.

(a) Use your equation from part 1(b) to complete the following table (use as many decimal places as possible):

Time interval	Change in height	Change in time	Average velocity
	Symbol:	Symbol:	Symbol:
	Units:	Units:	Units:
2.49 to 2.5 sec			
2.499 to 2.5 sec			
2.4999 to 2.5 sec			
2.5 to 2.51 sec			
2.5 to 2.501 sec			
2.5 to 2.5001 sec			

(b) What value do the average velocity values seem to be approaching? What does that limiting value represent in practical terms? What does that value represent in geometrical terms? Illustrate that value on your graph in part 1(c).

(c) In general how can you express the instantaneous rate of change of $f(x)$ at $x = a$?